Composite Basket Model

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21st July 2004

In this paper, we propose a new methodology for quoting loss baskets (and potentially diversified FTD baskets) in a consistent manner that overcomes the problems associated with simple Gaussian copula models. We find that the model calibrates reasonably well to the 5yr and 10yr iTraxx Europe tranches, the 5yr and 10yr iTraxx North America tranches and the standardised, diversified FTD baskets currently quoted in the broker market. In light of these results, we postulate that the model should also be extensible to bespoke portfolios.

Gaussian Copula and Correlation Skew

The Gaussian copula approach has proven very popular in the pricing of basket tranche products. This success can mostly be attributed to the simplicity and efficiency of its implementation (a comparison may be drawn with the Black and Scholes model used in option pricing). However, the key inputs to the model are default correlations which are not directly observable. These parameters can be inferred from historical equity prices, asset swaps spreads or default swaps levels, but none of these is particularly satisfactory. Alternatively, a single (average) correlation for each tranche can be implied from market prices. Unfortunately, when correlations are implied from all of the tranche prices observed in the market, each tranche typically has its own individual correlation which is different from the others. Specifically, we find that the market charges relatively more for equity or senior tranches and much less for mezzanine tranches than this model implies. This is often explained in two ways. On one hand there is a supply and demand argument that investors are nervous of the risk inherent in equity tranches while mezzanine tranches are extremely popular with investors and sellers of protection on senior tranches seem to require a minimum coupon irrespective of subordination. On the other hand the Gaussian copula loss distribution underestimates the perceived chances of a very low or very high number of defaults whilst significantly over-estimating the chances of observing a few defaults.

In practice, the independent calibration of the Gaussian copula model to each individual tranche gives rise to a correlation "skew", "smile" or "smirk". Its existence presents two serious problems. The first problem is a conceptual one: the correlation parameters of the copula are asset specific quantities and not tranche specific ones. Therefore they ought to be the same irrespective of the tranche being priced. The second problem is more practical: the sensitivity of a tranche to the correlation parameters is strongly related to its position in the capital structure. While equity tranche spreads drop with an increase in correlation, very senior tranches behave in an inverse manner. In turn mezzanine tranches are not very sensitive to correlation at all. Figure 1 illustrates this aspect.

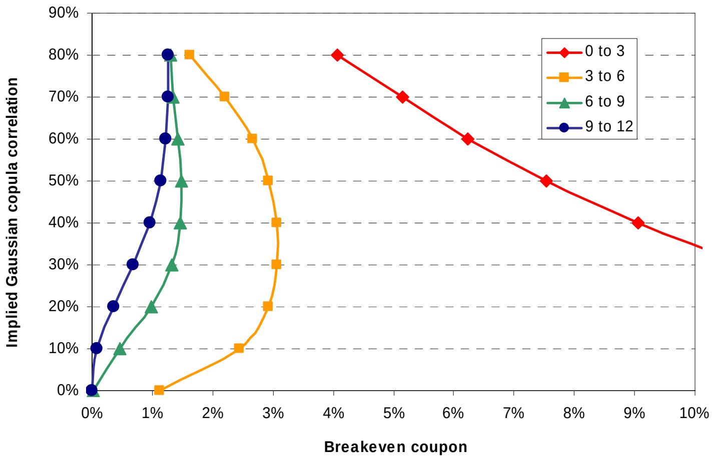

Figure 1: Implied Gaussian copula correlation vs. breakeven coupon for four of the iTraxx Europe tranches.

This fact creates a very serious problem for the pricing of tranches because even very small changes in the spread of a mezzanine tranche can cause very large swings in the corresponding correlation. In addition the correlation may not be unique (two different correlations may produce the same price) and in extreme cases it may not even be possible to find a correlation value that corresponds to the observed price.

Several approaches have been proposed to overcome this problem. For example, several authors have looked at the effect of assuming that some of the input parameters are random (see, for example, [1]). Others have tried alternative copulas [2] or stochastic intensity models (for example [3] and [4]). However none of these approaches has become widely accepted and the problem remains. Most notably an alternative calibration scheme was recently proposed where the calibration is applied to multiple first loss tranches covering the relevant part of the capital structure [5]. This approach relies on interpolation between published quotes and it is not clear how it can be generalised to the pricing of bespoke tranches.

Composite Basket Model

We propose an approach that relies on the commonly held intuition that investors allocate the risk in a given credit to different "buckets": some of the risk is driven by the particular sector, which can be a country and/or an industry group; some is specific to the company in question; and some is driven by some form of market consensus on the general outlook of the economy. We then model the first two with independent Poisson processes (respectively the *systemic* and *idiosyncratic* processes) and the latter through the Gaussian copula (we will refer to this as the *copula* term). We named the method the *Composite Basket Model,* or CBM.

More formally, given the probability of survival of an asset, $p(t)$, and one systemic sector only, we write:

$$p_S(t)\,p_C(t)\,p_I(t) = p(t)\ ,$$

where $p_S(t)$, $p_I(t)$ and $p_C(t)$ are respectively the systemic, idiosyncratic and copula survival probabilities. At least in the case of a flat intensity curves we can interpret this as a linear combination of spreads:

$$\begin{aligned} &p_S(s_S,t)\,p_C(s_C,t)\,p_I(s_I,t) = p(s,t) \\ &\Leftrightarrow s_S + s_C + s_I = s \end{aligned} .$$

In the simple case of N assets with identical probabilities and unity loss amounts, the loss distribution, conditioned on the systemic and copula terms, X_s and X_c respectively, is:

$$\begin{aligned} &\mathrm{P}(l \mid X_S\ X_C) = \\ &{}^N C_l \left[1 - p(s \mid X_S\ X_C)\right]^l p(s \mid X_S\ X_C)^{N-l} \end{aligned} ,$$

where we dropped the time argument for simplicity of notation. The default probabilities then expand, as postulated by our model, into:

$$p(s \mid X_S\ X_C) = p(s_S \mid X_S)\,p(s_C \mid X_C)\,p(s_I)\ .$$

The systemic factor can generate only two states: default with probability p_S, and survival with probability $(1 - p_S)$. Integrating over X_S yields:

$$\begin{aligned} &\mathrm{P}(l \mid X_C) = (1 - p_S)1_{l,N} + \\ &p_S\,{}^N C_l \left[1 - p(s_C \mid X_C)p(s_I)\right]^l \left[p(s_C \mid X_C)p(s_I)\right]^{N-l} \end{aligned} ,$$

where $1_{l,N}$ is the indicator function. Finally, we can integrate over X_C in the usual Gaussian copula fashion. Given the density $g(X_c)$:

$$\mathrm{P}(l) = \int \mathrm{P}(l \mid X_C)\,g(X_C)\,dX_C\ .$$

As before, in the more generic case of multiple survival probabilities and loss amounts more complex calculation methods have to be employed. However, the conditioning steps still hold true, given that the probabilities are adjusted in the correct fashion, as shown above.

Given the loss density function $\mathrm{P}(l,p_1,\ldots p_N)$ of a basket of N independent assets with survival probabilities p_1, through p_N, the loss distribution conditioned on the copula factor becomes:

$$\begin{aligned} &\mathrm{P}(l \mid X_C) = (1 - p_S)1_{l,N} + \\ &p_S\mathrm{P}\left[l,\ p(s_{C1} \mid X_C)p(s_{I1}), \ldots,\ p(s_{CN} \mid X_C)p(s_{IN})\right] \end{aligned} .$$

We can generalise these expressions quite easily to multiple systemic sectors.

CALIBRATION & RESULTS

We chose to use a fixed idiosyncratic intensity (floored as required to ensure that the probabilities are not negative) and two separate systemic intensities for the US and EU markets. We then calibrated these parameters to fit two different sets of quotes: DJ.CDX.NA.2 (in this note referred to as *iTraxx North America*) and iTraxx Europe 5 year tranches, as of the 14th of July 2004 (10 data points). The calibration was configured to favour results that fall inside or closer to the bid/offer range but to select weakly inside that range. The parameters are listed in table 1 and the two data sets in tables 2 and 3.

The equity tranches are quoted in terms of an upfront fee and a fixed running coupon of 5% in all cases. These were converted into an effective coupon (with no upfront fee) using the standard Gaussian copula model with a fitted single correlation parameter.

If the model result lies outside the bid/offer range, the errors shown are relative errors to the closest quote (negative if below bid, positive if above offer). If inside this range, no error is reported. These error values are shown only to assist in comparing results. During the calibration a suitable error measure was used.

The idiosyncratic intensity shown is a maximum value. For each asset the actual intensity used will be capped in such a way that the survival probabilities of each asset are correctly calibrated to the single name market.

Table 1: Composite basket model parameters

US systemic intensity	5.6bp
EU systemic intensity	9.0bp
Idiosyncratic intensity	32bp
Copula correlation	44%

Table 2: iTraxx Europe 5y as of the 14 July 2004 (average spread 46bp). Equity tranche (0-3) spreads correspond to the effective spread equivalent to an upfront payment of 30/32.5%.

Tranche	Bid	Offer	CBM	Error
12 – 22	0.21%	0.26%	**0.21%**	-2%
9 – 12	0.50%	0.57%	**0.45%**	-11%
6 – 9	0.82%	0.90%	**0.96%**	+6%
3 – 6	1.95%	2.10%	**2.14%**	+2%
0 – 3	13.69%	14.57%	**13.95%**	--

Table 3: iTraxx North America 5y as of the 14 July 2004 (average spread 62bp). Equity tranche (0-3) spreads correspond to the effective spread equivalent to an upfront payment of 40/42%.

Tranche	Bid	Offer	CBM	Error
15 – 30	0.13%	0.17%	**0.17%**	--
10 – 15	0.44%	0.54%	**0.66%**	+22%
7 – 10	1.31%	1.38%	**1.31%**	--
3 – 7	3.43%	3.65%	**3.44%**	--
0 – 3	17.82%	18.70%	**18.61%**	--

We then applied the same set of parameters to the corresponding 10 year quotes and to a second set of US quotes corresponding to a different date. These are shown in tables 4 through 7.

Table 4: iTraxx Europe 10y as of 14 July 2004 (average spread 65bp). Equity tranche (0-3) spreads correspond to the effective spread equivalent to an upfront payment of 49.7/54.7%

Tranche	Bid	Offer	CBM	Error
12 – 22	0.70%	0.90%	**0.68%**	-3%
9 – 12	1.31%	1.61%	**1.63%**	+1%
6 – 9	2.08%	2.48%	**2.45%**	--
3 - 6	4.57%	5.17%	**4.89%**	--
0 – 3	15.98%	17.79%	**16.64%**	--

Table 5: iTraxx North America 10y as of 14 July 2004 (average spread 83bp). Equity tranche (0-3) spreads correspond to the effective spread equivalent to an upfront payment of 56.5/60%.

Tranche	Bid	Offer	CBM	Error
15 - 30	N/A	N/A	**0.57%**	N/A
10 - 15	1.69%	1.93%	**1.82%**	--
7 - 10	3.10%	3.45%	**2.98%**	-4%
3 - 7	6.45%	7.05%	**6.22%**	-4%
0 – 3	19.32%	20.90%	**20.81%**	--

Table 6: iTraxx North America 5y as of 26 May 2004 (average spread 66bp). Equity tranche (0-3) spreads correspond to the effective spread equivalent to an upfront payment of 44.5/45.5%.

Tranche	Bid	Offer	CBM	Error
15 – 30	0.12%	0.20%	**0.20%**	--
10 – 15	0.53%	0.68%	**0.79%**	+16%
7 – 10	1.46%	1.57%	**1.44%**	-1%
3 – 7	3.70%	4.05%	**3.83%**	--
0 – 3	19.61%	20.08%	**19.38%**	-1%

Table 7: iTraxx North America 10y as of 26 May 2004 (average spread 88bp). Equity tranche (0-3) spreads correspond to the effective spread equivalent to an upfront payment of 57/67%.

Tranche	Bid	Offer	CBM	Error
15 - 30	N/A	N/A	**0.66%**	N/A
10 - 15	1.70%	2.20%	**1.99%**	--
7 - 10	3.45%	4.00%	**3.32%**	-4%
3 - 7	7.25%	8.05%	**6.66%**	-8%
0 – 3	19.53%	25.08%	**21.52%**	--

We find a reasonable agreement between the model results and the published quotes across different markets and different dates. These results are much better than is possible using the Gaussian copula. Naturally the results obtained are preliminary as there is limited data with which to test the method. Using historical data, we observe only small variations in underlying spreads for the period in which reliable data can be obtained. Further work will be required to establish the model stability through time. However, the demonstrated ability to match prices for the European and US markets (which have fairly different average spreads) is significant and suggests that the model would display some robustness with respect to movements in the underlying spreads. Additionally, a similar conclusion can be drawn from the model's ability to match prices across the five and ten year maturities.

The comparisons yield one other interesting result. If we split the systemic risk into Europe and the US then the natural interpretation for this value is that it captures the default risk inherent in that particular economy. The argument then implies that the fitted intensities should be comparable to market quotes on sovereign debt. Quotes on government debt are approximately 0.5-2bp for the US and 3-7bp for Europe. On the other hand, assuming a recovery of 30%, the systemic intensities correspond to spreads of 3bp and 6bp respectively for the US and Europe. These are in reasonable agreement with the quoted values and raise the possibility of reducing the number of free parameters and calibrating the systemic terms independently of the baskets.

It is also of interest to note that these results seem to be applicable to the first-to-default market. Restricting ourselves to the four FTD standardised baskets (launched recently) that are generally seen as uncorrelated both competitively and geographically then:

Table 8: First-to-default baskets as of 14 July 2004.

Basket	Bid	Offer	CBM	Error
Diversified	2.83%	3.16%	**2.84%**	--
Energy	1.46%	1.66%	**1.59%**	--
Consumer-noncyclical	2.10%	2.35%	**2.18%**	--
Cross-over	6.62%	7.29%	**6.43%**	-3%

Further work is required to establish whether this first-to-default result is meaningful. However, the fact that parameters fitted to tranche instruments seem also to apply in the instances shown above is by itself a very interesting observation.

In this paper we focussed on value calculations but first order risks are equally critical aspects of derivative trading. Although this is not shown in the results above, we have compared the credit spread deltas produced by the Composite Basket Model against those produced by the Gaussian copula calibrated to first-loss tranches (i.e. using base correlations as discussed in [5]). We found the results to be very similar for the most traded mezzanine tranches. Equity tranches exhibit slightly less leverage and senior tranches more leverage in the CBM than is obtained with the base correlation approach.

CONCLUSIONS

We introduced a new proposal for dealing with the Gaussian copula skew and the pricing of bespoke baskets in the single tranche market. The Composite Basket Model acknowledges the fact that the Gaussian copula or similar methods are widely used by market participants to price these products while it also attempts to rationalise how market participants adjust their prices to accommodate their own views of risk.

The results presented indicate that this approach is capable of capturing much of the structure implied by the published quotes: once the parameters are calibrated, the model fits the 5yr and 10yr iTraxx Europe, the 5yr and 10yr iTraxx North America and the standardised FTD baskets. These results, obtained for different terms and valuation dates, indicate that a trivial generalisation to bespoke baskets is possible with this method at least as long as our baskets are as diversified as the indices themselves are.

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